

# ON CONSISTENCE OF MATERIAL COUPLING IN A $GL(3,R)$ GAUGE FORMULATION OF GRAVITY

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A covariant scheme for material coupling with  $GL(N, R)$  gauge formulation of gravity is studied. We revisit a known idea of a Yang-Mills type construction, where quadratical power of cosmological constant have to be considered in consistence with vacuum Einstein's gravity. Then, matter coupling with gravity is introduced and some constraints on fields and background appear. Finally, exploring the  $N = 3$  case we elucidate that introduction of auxiliary fields decreases the number of these constraints.

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The old problem about construction of gauge theory for gravitation cover a large list of approaches (among others, see some fundamental references<sup>1-9</sup>). Starting with Utiyama<sup>1</sup> who was the first to recognize the "gauge" character of gravitational field, passing by constructions which relax symmetry property of connection (i.e., Riemann-Cartan spacetime) and others where metric-compatible condition is also removed, arising theories based on non-Riemannian geometries<sup>8,9</sup>.

However, in order to compare and to explore possible inconsistencies between solutions coming from Einstein's theory and those of a gauge formulation of gravity, here we focus our attention in a construction based on a subgroup of the affine<sup>8,9</sup>, this means  $GL(N, R)$  as gauge group<sup>5</sup>. Obviously, choosing this subgroup causes limitations related to the context of supersymmetric theories which demand translation symmetries, as an example. But having in mind to show inconsistencies, it is sufficient the aforementioned construction in a Riemannian spacetime.

It is well known that a  $GL(N, R)$  Yang-Mills type lagrangian density will be related to a purely quadratic lagrangian density on Riemann-Christoffel curvature. These lagrangians take great interest, because, from a point of view of standard field theory these yield theories where the renormalization problems are much less severe<sup>10</sup>; from the string theory point of view, this class of terms does appear at low energy limit of effective lagrangian for gravity<sup>11</sup>, among other things.

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The aim of this letter is to explore a general and covariant scheme for non minimal coupling of material fields in  $N$  dimension with  $GL(N, R)$  connection. So, we study consistence between solutions coming from this type of theory and the Einstein's gravity ones. At the trivial vacuum case, last requirement means to consider quadratical power of cosmological constant.

On the other hand, when material fields are turned on, it is observed that consistence gives rise to constraints on these fields and background (i.e., cosmological constant). The last step consists of including action terms dependent on auxiliary fields, showing that some restrictions can be avoided. As a useful scenery to explore and illustrate coupling we will study the particular case in  $2 + 1$  dimension.

A brief review about notation is presented. Let  $V_\mu^a$  be the components of a tensorial object with curved and lorentz indexes, defined in an  $N$  dimensional space provided with metric  $g_{\mu\nu}$ , curved coordinates  $x^\mu$ , with  $\mu, \nu, \dots = 0, 1, \dots, N - 1$  and locally flat  $\xi^a$ , with  $a, b, c, \dots = 0, 1, \dots, N - 1$  (the Minkowski metric is  $\eta_{ab} = \text{diag}(-1, +1, \dots, +1)$ ). Then, introducing the spin (i.e.,  $\omega_{\mu b}^a$ ) and affine (i.e.,  $\Gamma_{\mu\nu}^\lambda$ ) connections, the well known covariant derivative is  $D_\mu V_\nu^a = \partial_\mu V_\nu^a + \omega_{\mu b}^a V_\nu^b - \Gamma_{\mu\nu}^\lambda V_\lambda^a$ , etc. Particularly, property  $D_\mu e_\nu^a = 0$  is taken, where  $e_\nu^a$  is the  $n$ -bein which satisfies  $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$ . From this, follows that the torsion can be written in the form

$$T_{\mu\nu}^\lambda \equiv \Gamma_{\mu\nu}^\lambda - \Gamma_{\nu\mu}^\lambda = e_\mu^a (\partial_\nu e_\nu^a - \partial_\nu e_\mu^a + \omega_{\mu\nu}^a - \omega_{\nu\mu}^a) , \quad (1)$$

where,  $\omega_{\mu\nu}^a \equiv e_\nu^b \omega_{\mu b}^a$ , etc.

If the matrix elements for  $GL(N, R)$  and lorentz transformations are defined as  $(U)^\alpha_\mu \equiv \frac{\partial x'^\alpha}{\partial x^\mu}$  and  $(L)^a_b \equiv \frac{\partial \xi'^a}{\partial \xi^b}$ , covariant behavior of derivative  $D_\mu$  demands following transformation rules for connections

$$\omega'_\mu = L \omega_\mu L^{-1} + L \partial_\mu L^{-1} , \quad (2)$$

$$A_a' = U A_a U^{-1} + U \partial_a U^{-1} , \quad (3)$$

where we have introduced notation

$$(A_a)^\mu_\nu \equiv e^\alpha_a \Gamma^\mu_{\alpha\nu} . \quad (4)$$

It can be observed that the object (4) is a  $GL(N, R)$  connection which transforms like a lorentzian vector in flat index. So,  $GL(N, R)$  is chosen as the structure group, and will be assumed that the fibre projection, the transition functions, etc. are given.

The Riemann-Christoffel curvature tensor (i.e.,  $R^\alpha_{\sigma\mu\nu}$ ) can be constructed through application of commutator  $[D_\mu, D_\nu] \equiv D_\mu D_\nu - D_\nu D_\mu$  on a rank one tensor, in other words

$[D_\mu, D_\nu]V^\alpha = R^\alpha_{\sigma\nu\mu}V^\sigma - T^\lambda_{\mu\nu}D_\lambda V^\alpha$ , for all  $V^\alpha$  implies  $R^\sigma_{\alpha\mu\nu} = \partial_\nu\Gamma^\sigma_{\alpha\mu} - \partial_\mu\Gamma^\sigma_{\alpha\nu} + \Gamma^\lambda_{\alpha\mu}\Gamma^\sigma_{\lambda\nu} - \Gamma^\lambda_{\alpha\nu}\Gamma^\sigma_{\lambda\mu}$ . Using this definition, the Riemann-Christoffel tensor components can be written in the form

$$R^\sigma_{\alpha\mu\nu} = e_\mu{}^a e_\nu{}^b (F_{ab})^\sigma{}_\alpha, \quad (5)$$

where

$$(F_{ab})^\sigma{}_\alpha \equiv (D_b A_a - D_a A_b + [A_a, A_b] + T^c{}_{ab} A_c - T^c{}_{ab} T_c)^\sigma{}_\alpha, \quad (6)$$

is a Yang-Mills like curvature with torsion contribution. Notation means  $(T_c)^\sigma{}_\alpha \equiv e_{\mu c} g^{\sigma\nu} T^\mu{}_{\nu\alpha}$ , etc.

In order to study the relationship between solutions obtained from a Yang-Mills type lagrangian formulation defined with curvature  $F_{ab}$ , and solutions of Einstein's gravity with cosmological constant in a Riemannian space ( $EG\lambda$ ), we will consider that Ricci tensor satisfies the field equation of Einstein (i.e.,  $R^{\alpha\beta} - \frac{g^{\alpha\beta}}{2}R - \lambda g^{\alpha\beta} = -8\pi G T^{\alpha\beta}$ , where  $T^{\alpha\beta}$  is the energy-momentum tensor associated to material fields and  $\lambda$  is the cosmological constant). This ends the review.

We proceed noting the formulation without matter. Let the gauge invariant action for  $T^{\alpha\beta} = 0$  be

$$S_o = \int d^N x \sqrt{-g} \left( -\frac{1}{4} \text{tr} F^{ab} F_{ab} + \Lambda \right), \quad (7)$$

where  $\Lambda$  would be related to cosmological constant. Observe that action (7) matches with a standard Riemann-Christoffel quadratic lagrangian theory in a torsionless space-time. In dynamical analysis we will assume a Palatini's variational principle type (i.e., variations on  $GL(N, R)$  connection and  $n$ -bein) thinking about a general case where  $T^\lambda_{\mu\nu} \neq 0$ . Afterwards, one can evaluate field equations in a particular space-time. An alternative starting point would be to consider a standard lagrange multiplier method, where the torsionless condition works as constraint on connection, and can be obtained the same physical results that we show in next.

Functional variation on connection  $A$  gives

$$\delta_A S_o = \int d^N x \sqrt{-g} \text{tr} E^\sigma \delta A_\sigma, \quad (8)$$

up to a boundary term. Notation means  $(E^\sigma)^\lambda{}_\alpha \equiv (\frac{1}{\sqrt{-g}} \partial_\mu(\sqrt{-g} F^{\lambda\mu}) + [F^{\mu\lambda}, A_\mu - T_\mu])^\sigma{}_\alpha$ , and in a torsionless space-time must be  $(E^\sigma)^\lambda{}_\alpha = D_\mu(F^{\lambda\mu})^\sigma{}_\alpha$  (i.e., field equation is  $D_\mu F^{\lambda\mu} = 0$ ). This last relation can be written in terms of Ricci tensor with the help of Bianchi's identities in the form

$$(E_\sigma)_{\lambda\alpha} = D_\alpha R_{\lambda\sigma} - D_\sigma R_{\lambda\alpha}. \quad (9)$$

From this, the torsionless solutions for  $\delta_A S_o = 0$  are those of de Sitter and Anti de Sitter (dS/AdS) type. Then, taking the trivial solution  $R_{\alpha\beta} = -(2\lambda/(N-2))g_{\alpha\beta}$  (unique solution under this conditions), the curvature  $F_{ab}$  can be evaluated using eq.(5):

$$(F_{ab})^\alpha{}_\mu = \frac{2\lambda}{(N-2)(N-1)}(e^\alpha{}_b e_{\mu a} - e^\alpha{}_a e_{\mu b}) . \quad (10)$$

Next we consider  $n$ -bein's variation on action  $S_o$ , obtaining the field equation

$$tr F_{db} F_c{}^b - \frac{1}{4} \eta_{dc} tr F^{ab} F_{ab} + \Lambda \eta_{dc} = 0 , \quad (11)$$

and demanding consistence with eq.(10) it says that

$$\Lambda = -\frac{2(N-4)}{(N-2)^2(N-1)}\lambda^2 . \quad (12)$$

In other words, this condition guarantees that dS/AdS are trivial solutions for extremal of action (7). It can be noted that constant  $\Lambda$  does not distinguish between dS or AdS in this lagrangian formulation in contrast with the Hilbert-Einstein's one. However,  $\Lambda$  establishes other type of classification. When  $N = 3$ ,  $\Lambda$  takes the value  $\lambda^2 \geq 0$ , while cosmological constant does not explicitly appear in the action for  $N = 4$ . On the other hand, if  $N > 4$  one have  $\Lambda \leq 0$ . Immediately one can say that the physical content of these classes of  $\Lambda$  are related to the shift on the Hamiltonian.

The model for coupling with matter through the connection  $A_a$  is outlined. We explore a possible general covariant scheme for non minimal coupling without auxiliary fields in the following manner

$$S = S_o + \int d^N x \sqrt{-g} (\ell(e, \psi) + 4\pi G tr M^{ab}(e, \psi) F_{ab}) , \quad (13)$$

where gauge invariant functional  $\ell(e, \psi)$  and tensor  $M^{ab}(e, \psi)$  are functionals on  $n$ -bein and material fields. From these definitions, it follows that, after an integration of action (13) it can be obtained “current” (i.e., minimal coupling) and “mass” (i.e., Proca type coupling) terms. Now, our problem is to explore the shape of objects  $\ell(e, \psi)$  and  $M^{ab}(e, \psi)$  requiring consistence with Einstein's solutions.

Here, we propose

$$(M^{\alpha\beta})^\mu{}_\nu = (N^{\alpha\beta\sigma\rho})^\mu{}_\nu T_{\sigma\rho} + (n^{\alpha\beta})^\mu{}_\nu , \quad (14)$$

where objects  $N^{\alpha\beta\sigma\rho}$  and  $n^{\alpha\beta}$  depend only on metric. Their general form, in consistence with symmetry properties are

$$\begin{aligned} (N^{\alpha\beta\sigma\rho})^\mu{}_\nu \equiv & c_1 (g^{\mu\alpha} \delta^\beta{}_\nu g^{\sigma\rho} - g^{\mu\beta} \delta^\alpha{}_\nu g^{\sigma\rho}) + c_2 (g^{\rho\alpha} \delta^\beta{}_\nu g^{\sigma\mu} - g^{\rho\beta} \delta^\alpha{}_\nu g^{\sigma\mu}) \\ & + c_3 (g^{\mu\alpha} g^{\rho\beta} \delta^\sigma{}_\nu - g^{\mu\beta} g^{\rho\alpha} \delta^\sigma{}_\nu) , \end{aligned} \quad (15)$$

$$(n^{\alpha\beta})^\mu{}_\nu \equiv a(g^{\mu\alpha}\delta^\beta{}_\nu - g^{\mu\beta}\delta^\alpha{}_\nu) , \quad (16)$$

and  $c_1, c_2, c_3, a$  are real free parameters.

It is very important to note that, as an illustration in this first approach to a coupling scheme we consider a system whose energy-momentum tensor does not explicitly depend on connection, just for simplicity. One can think about a class of this type of systems. For example, the energy-momentum tensor takes the form

$$T_{\mu\nu} = (\alpha \delta^\lambda{}_\mu \delta^\rho{}_\nu + \beta g^{\lambda\rho} g_{\mu\nu}) \psi_{\lambda\rho} , \quad (17)$$

with  $\alpha$  and  $\beta$  real scalars, and  $\psi_{\lambda\rho}$  be a symmetric tensor containing information about matter fields. Tensor (17) can describe some interesting systems. If  $\alpha = -1$ ,  $\beta = \frac{1}{2}$  and  $\psi_{\lambda\rho} = \partial_\lambda \phi \partial_\rho \phi$  we are talking about a massless real scalar field  $\phi$ . On the other hand, if  $\alpha = p + \rho$ ,  $\beta = p$ , and  $\psi_{\lambda\rho} = U_\lambda U_\rho$ , we are considering a perfect fluid with density  $\rho$ , pressure  $p$  and velocity  $U_\lambda$ . In a similar way, taking suitable definitions can be included an electro-magnetic field or a bosonic string. Anyway, we assume that  $\psi_{\lambda\rho}$  does not depend on metric or on connection (maybe some situation, where  $\psi_{\lambda\rho}$  depends only on metric, would be considered, but the essentially physical results will not be much different).

So, performing  $\delta_A S$  variations, one obtains

$$\delta_A S = \int d^N x \sqrt{-g} \left( (E^\sigma)^\lambda{}_\alpha - 8\pi G \left( \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} (M^{\lambda\mu})^\sigma{}_\alpha) + [M^{\mu\lambda}, A_\mu - T_\mu]^\sigma{}_\alpha \right) \right) \delta(A_\sigma)^\alpha{}_\lambda , \quad (18)$$

up to a boundary term. When equation of motion arising from  $\delta_A S = 0$  is written in a torsionless space-time, it leads to

$$\begin{aligned} & D_\nu (R^{\alpha\mu} - 8\pi G c_1 g^{\alpha\mu} T - 8\pi G c_2 T^{\alpha\mu} + c_4 \lambda g^{\alpha\mu}) \\ & - D^\mu (R^\alpha{}_\nu - 8\pi G c_1 \delta^\alpha{}_\nu T - 8\pi G c_3 T^\alpha{}_\nu + c_4 \lambda \delta^\alpha{}_\nu) \\ & + 8\pi G (c_2 \delta^\alpha{}_\nu D_\beta T^{\mu\beta} - c_3 g^{\alpha\mu} D_\beta T^\beta{}_\nu) = 0 , \end{aligned} \quad (19)$$

where we had made use of freedom to introduce cosmological terms with parameter  $c_4$ . Equation (19) says that  $EG\lambda$  is still trivial solution (our consistence condition), if parameters take the following values

$$c_4 = 2c_1 = \frac{2}{N-2} , \quad c_2 = c_3 = -1 . \quad (20)$$

But, as we will show, there are more restrictions on material fields, this time when  $n$ -bein variations are performed on action  $S$ . Obviously, we need to say something about the shape

of  $\ell(e, \psi)$ . On one hand, this lagrangian density must be consistent with vacuum limit of the theory ( $\ell(e, \psi) \rightarrow 0$  if  $\alpha$  and  $\beta$  go to zero). On the other hand, we demand consistency with a *limit of no gravitational coupling*, which consists of performing the limit  $G \rightarrow 0$  and  $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$  with  $\lambda = 0$ . Under these conditions, action (13) must correspond to the free flat theory of material fields. With this in mind, we propose a general form for material contribution:

$$\ell(e, \psi) \equiv L(\psi) + b_1 T^2 + b_2 T_{\mu\nu} T^{\mu\nu} = q + k \psi + b \psi^2 , \quad (21)$$

where  $L(\psi) = k_{(\alpha, \beta)} \psi + q_{(\alpha, \beta)}$  is the lagrangian density of considered matter fields in a curved background and  $b = b_1(\alpha + N\beta)^2 + b_2(\alpha^2 + 2\alpha\beta + N\beta^2)$  is a real parameter. Later we study the limit of no gravitational coupling for parameter  $b$ .

Then, variations of  $n$ -bein in action (13) can be written in terms of Ricci, Weyl and material tensors. So, the field equation is

$$P^\sigma_d [\psi_{\alpha\beta}, e^\mu_b, R_{\mu\nu}] + Q^\sigma_d [e^\mu_b, R_{\mu\nu}] + S^\sigma_d [\psi_{\alpha\beta}, e^\mu_b, R_{\mu\nu}, C_{\mu\nu\alpha\beta}] = 0 , \quad (22)$$

where  $P^\sigma_d$  and  $Q^\sigma_d$  are quadratical polynomial functionals on  $\psi_{\alpha\beta}$  and Ricci tensor, respectively. Moreover,

$$\begin{aligned} S_{\alpha\beta} \equiv & C_{\mu\nu\lambda\alpha} C^{\mu\nu\lambda}_\beta - \frac{g_{\alpha\beta}}{4} C_{\mu\nu\rho\lambda} C^{\mu\nu\rho\lambda} - C_{\mu\nu\lambda\alpha} R^{\mu\nu\lambda}_\beta - C_{\mu\nu\lambda\beta} R^{\mu\nu\lambda}_\alpha \\ & + \frac{g_{\alpha\beta}}{2} C_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda} - 16\pi G \alpha C_{\alpha\lambda\beta\rho} \psi^{\lambda\rho} , \end{aligned} \quad (23)$$

where  $C_{\mu\nu\lambda\alpha}$  is the Weyl tensor. At this point, one can explore the particular case in which  $N = 3$  because the Weyl tensor is null identically. Then, if one expect that action (13) must be an extremal on  $EG\lambda$  we evaluate equation of motion for dreibein (22)

$$\begin{aligned} & (8\pi G)^2 \left( -\frac{4b}{(8\pi G)^2} + 4\alpha^2 - 26\alpha\beta + 2\beta^2 \right) \psi \psi^\sigma_d \\ & + (8\pi G)^2 \left( \frac{b}{(8\pi G)^2} - \alpha^2 - 4\alpha\beta - 7\beta^2 \right) \psi^2 e^\sigma_d \\ & + 8\pi G \left( (22\alpha - 12\beta)\lambda - 16\pi G \alpha a - \frac{k}{4\pi G} \right) \psi^\sigma_d \\ & + 8\pi G \left( (2\alpha - 8\beta)\lambda - 16\pi G \beta a - \frac{k}{8\pi G} \right) \psi e^\sigma_d \\ & + (q + 16\pi G a \lambda) e^\sigma_d = 0 . \end{aligned} \quad (24)$$

The last equation of motion necessarily represents a restriction for material fields. Therefore the following restriction arise

$$\psi = \text{constant} \epsilon R_e , \quad (25)$$

on possible field configurations. This is not severe in case of a perfect fluid (i.e.,  $\psi = U^\mu U_\mu = -1$ ). Anyway, for all  $\psi^\sigma_d$  with  $\psi = \text{constant}$ , equation (24) gives two relations for  $a$  and  $b$

$$2\psi b + (8\pi G)^2 \alpha a = (8\pi G)^2 (2\alpha^2 - 13\alpha\beta + \beta^2) \psi + 8\pi G (11\alpha - 6\beta) \lambda - k , \quad (26)$$

$$\psi^2 b + 16\pi G (\lambda - 8\pi G \beta \psi) a = (8\pi G)^2 (\alpha^2 + 4\alpha\beta + 7\beta^2) \psi^2 - 8\pi G (2\alpha - 8\beta) \lambda \psi - k \psi - q . \quad (27)$$

In order to obtain regular solutions one need to demand the new restriction

$$\psi(4\lambda - 8\pi G(\alpha + 4\beta)\psi) \neq 0 . \quad (28)$$

With the last condition one can study the limit at no gravitational coupling for  $b$

$$b|_{\lambda=0} = (8\pi G)^2 \left( \frac{\alpha^3 + 8\alpha^2\beta - 26\alpha\beta^2 + 9\beta^3}{\alpha + 4\beta} \right) - \frac{(\alpha + 2\beta)k\psi + q\alpha}{(\alpha + 4\beta)\psi^2} , \quad (29)$$

being consistent in case of a massless scalar field, in which  $\alpha = -1$ ,  $\beta = 1/2$  and  $q = 0$  (i.e., when  $G \rightarrow 0$ , relation (29) gives  $b|_{\lambda=0} = (8\pi G)^2(\dots) \rightarrow 0$ ). This is not the same situation for a fluid, because the limit  $b|_{\lambda=0} \rightarrow 0$  demands an additional restriction on density and pressure:  $(p + \rho)p = 0$  (i.e., one should consider dust).

We want to underline that eq.(25) and eq.(28) means that non minimal coupling scheme presented in eq.(13) is consistent with Einstein gravity only under certain conditions related with the class of material distribution and the features of space-time (i.e., restrictions on possible values of cosmological constant). This is not a surprising idea. In fact, from the point of view of high spin gauge fields coupled to gravity it can be found that theory is consistent only in restricted backgrounds<sup>12</sup>. So, maybe new interaction terms (i.e., involving auxiliary fields ) added to action (13) enclose the hope to reduce the number of constraints on material fields.

In this sense, let  $S'$  be the new action with  $J^a$  and  $H^{\alpha\beta}$  some functional on dreibein and material fields, and  $(W_a)^\mu{}_\nu$  the components of a auxiliary field, which transforms like a  $GL(3, R)$  connection, then we write

$$\begin{aligned} S' = S_o + \int d^3x \sqrt{-g} & (\ell(e, \psi) + 4\pi G \text{tr} M^{ab}(e, \psi) F_{ab} \\ & + \text{tr} J^a (A_a - W_a) + \text{tr} H^{ab} (A_a - W_a)(A_b - W_b)) , \end{aligned} \quad (30)$$

where a naive proposal for components of  $J^a$  and  $H^{ab}$  is taken

$$(J_\beta)_{\mu\nu} \equiv (d_1 + d_2\psi)\varepsilon_{\beta\mu\nu} , \quad (31)$$

$$(H^{\alpha\beta})^{\mu\nu} \equiv a_1 g^{\alpha\beta} g^{\mu\nu} + a_2 g^{\alpha\mu} g^{\beta\nu} + a_3 g^{\alpha\nu} g^{\beta\mu} , \quad (32)$$

with the real parameters  $d_1$ ,  $d_2$ ,  $a_1$ ,  $a_2$  and  $a_3$ .

From eq.(30), equation of motion for  $W_b$  is

$$J^b + H^{ab}(A_a - W_a) + (A_a - W_a)H^{ba} = 0 , \quad (33)$$

establishing that field equations of connection fields  $A_a$  are maintained with or without introducing auxiliary terms. On the other hand, suggests an ansatz for auxiliary fields:

$$(A_\alpha - W_\alpha)_{\mu\nu} = (\theta_1 + \theta_2 \psi) \varepsilon_{\alpha\mu\nu} , \quad (34)$$

with  $\theta_1$  and  $\theta_2$  real parameters (this is not the most general linear dependence on field  $\psi_{\sigma\beta}$ , but it is sufficient and consistent with eq.(33)). Ansatz (34) give the relations  $d_1 = 2(a_2 - a_1)\theta_1$  and  $d_2 = 2(a_2 - a_1)\theta_2$ .

Next, equation of motion for dreibein is evaluated on  $EG\lambda$

$$\left( P^\sigma_d [\psi_{\alpha\beta}, e^\mu_b, R_{\mu\nu}] + Q^\sigma_d [e^\mu_b, R_{\mu\nu}] + tr \frac{\delta J^\beta}{\delta e_\sigma^d} (A_\beta - W_\beta) + tr \frac{\delta H^{\alpha\beta}}{\delta e_\sigma^d} (A_\alpha - W_\alpha) (A_\beta - W_\beta) - tr H^{\alpha\beta} (A_\alpha - W_\alpha) (A_\beta - W_\beta) e^\sigma_d \right)_{EG\lambda} = 0 . \quad (35)$$

Then, using eq.(31), eq.(32) and eq.(34), it can be found for any material field configuration an equation system for free parameters

$$16\pi G a \lambda - 14(a_1 - a_2)\theta_1^2 = -q_{(\alpha,\beta)} , \quad (36)$$

$$2(8\pi G)^2 \beta a + 12(a_1 - a_2)\theta_1 \theta_2 = 8\pi G (2\alpha - 8\beta) \lambda + k_{(\alpha,\beta)} , \quad (37)$$

$$(8\pi G)^2 \alpha a - 12(a_1 - a_2)\theta_1 \theta_2 = 8\pi G (11\alpha - 6\beta) \lambda - k_{(\alpha,\beta)} , \quad (38)$$

$$b - 14(a_1 - a_2)\theta_2^2 = (8\pi G)^2 (\alpha^2 + 4\alpha\beta + 7\beta^2) , \quad (39)$$

$$2b - 12(a_1 - a_2)\theta_2^2 = (8\pi G)^2 (2\alpha^2 - 13\alpha\beta + \beta^2) . \quad (40)$$

Consistence with limit at no gravitational coupling for  $b$  can be verified and, furthermore only for free parameters  $a$ ,  $b$ ,  $(a_1 - a_2)\theta_1^2$ ,  $(a_1 - a_2)\theta_2^2$  and  $\lambda$ , this system can be solved. From this, just restrictions on possible values for cosmological constant remains.

We finalize, saying that although the coupling terms presented in eq.(30) (with eq. (31) and (32)) do not have the most general form, the idea that strong restrictions on  $\psi_{\alpha\beta}$  can be avoided with auxiliary fields has been elucidated. A possible extension of this type of study pointing to metric-affine gravity, including non-Riemannian interaction terms<sup>9</sup>, must be performed elsewhere.

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